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PROFESSOR CAYLEY, F.R.S., President, in the Chair.

Charles Becker, Esq., 112 St. Martin's Lane;
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Ralph Copeland, Ph.D., Parsonstown Observatory, Ireland;
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Major W. Palmer, R.E., Southborough, Tunbridge Wells;
Dr. J. C. Robertson, Monaghan, Ireland;
Capt. F. C. B. Robinson, R.N., Junior United Service Club; and
Rev. Thos. Thistlethwaite Smith, Thruxton Rectory, Hereford,

were balloted for and duly elected Fellows of the Society.

Dr. Edouard Heis, Münster;
Dr. Theodor Oppolzer, Vienna;
Dr. Johann Freidrich Julius Schmidt, Athens Observatory;
and Mons. C. Wolf, Paris Observatory,

were also balloted for and duly elected Associates of the Society.

On a proposed New Method of Treating the Lunar Theory. By Sir George Biddell Airy, K.C.B., Astronomer Royal.

It appears desirable to introduce the explanation of the method now proposed by a rapid survey of the methods hitherto employed.

In the whole range of physical mathematics, there is perhaps nothing more remarkable than the beauty of the geometrical

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integrations, in the III. Book of *Newton's Principia*, for the Lunar Inequalities in Latitude and for the Lunar Variation; and the general accuracy of the results. It is clear also, from a few remarks in the 11th Section of the I. Book, and from an unexplained remark on the comparison of the inequalities of the satellites of other planets with those of our Moon in the III. Book, that Newton perfectly understood the origin of what are now called the terms of the second order, by which the Velocity of Progression of the Apse, and the Evection, are so much increased. But Newton published no numerical calculation of those quantities; and the theory was, so far, left imperfect. A more powerful calculus was necessary.

The want was supplied by the Differential Calculus, in the shape in which it was established among Continental mathematicians; and the particular form in which it was applied by Clairaut to the Lunar Theory exhibited at once the power of the Calculus and the ease of applying it. The simple form of Clairaut's differential equations for parallax and latitude opened out the entire process of extending the theory to any degree of accuracy, and showed at the same time the steps by which periodical inequalities of one form are deduced from the combination of periodical inequalities of other forms. I think that scarcely sufficient honour has been given to Clairaut for the formation of this special equation, without which the progress of the theory would probably have been very slow. Even now it is the best form in which a beginner can enter upon the studies of the Lunar Theory. Clairaut's theory gives the time in terms of the arc of longitude described, which is not without advantage in the treatment of equations of long period; but it requires a final reversion of series, in order to give the longitude in terms of the time. Mathematicians in the later part of the present century have preferred a form in which the Moon's ordinates are expressed immediately in terms of the time. I give my adhesion to this method; but at the same time I am anxious to offer my testimony to the value of the process so successfully introduced at a most critical point in the progress of the science.

The next important extensions of the theory were those of Laplace and Damoiseau: both founded on Clairaut's equation; both exhibiting the subordinate equations derived from the comparison of coefficients which are expressed by unexpanded algebraical fractions whose denominators are very complicated (the piles of these fractions, especially in Damoiseau's work, are appalling); both giving the first results in numerical values for the coefficients of numerous arguments which are multiples of longitude; both leaving in great obscurity the process by which the numerical solutions of these algebraical comparisons were obtained; and both giving the final results in terms depending on the time. Damoiseau, however, added to this investigation a work which demands our gratitude: a system of Lunar Tables expressly founded on the aggregation of simple periodical terms

having for arguments different multiples of the time. It was by use of these Tables (with small additions derived principally from Plana) that I conducted the great Reduction of Lunar Observations from 1750 to 1853, and deduced from them the corrections of the principal coefficients. Damoiseau's angular values were all expressed in the centesimal division of the quadrant: a method which possesses so many advantages that I hope for its adoption in future tables.

Plana's work, which followed, was not entirely pure in its method. It commences, for instance, with an application of theorems for the "variation of constants," here introduced with great advantage. But in the more advanced parts it may be described as established on the use of the time as the independent variable, and as exhibiting every coefficient in a series of algebraical terms without denominators. Viewed as leading to an algebraical result, this work was a great advance beyond all which had preceded it; and in numerical accuracy it is probable that something was gained.

I do not advert to the extensive investigations of Lubbock, because they were principally in the nature of verifications, adopting generally M. Plana's system. Nor do I consider the important questions raised by Professor Adams, because they are, in fact, a re-examination of specific points in a received theory. Professor Hansen's theory and tables require mention, principally in explanation of my reasons for almost omitting them from a view of the progress of the science. I attach the highest value to Professor Hansen's discovery of two inequalities in longitude produced by *Venus*; of which one is universally accepted, and the other, though controverted, still appears plausible. And I value the new equation which he introduced in the Moon's latitude. I believe also that the object which Professor Hansen originally proposed to himself, namely, the more rapid convergence of terms, has been (in some measure at least) attained. Yet I think that the general form of his theory, differing so much from the two systems which had preceded it, and presenting little facility for correcting elements from observations, is so far objectionable that it is not likely to be adopted by future lunar theorists; and that its introduction was, in fact, a retrograde step. But, in common with all who are practically concerned with lunar observations, I am grateful for his Lunar Tables, which, embodying the results of his own theory and the Greenwich corrections of elements, and published at a time when the existing tables were running wild, have been most beneficial to practical science.

But there remains one glorious work, almost superhuman in its labour, and perfect beyond others in the detailed exhibition of its results; the Lunar Theory of Delaunay. In this the time is adopted as the independent variable. The masses of undeveloped fractions here exhibited are greater than those of Damoiseau; the development in terms without denominators is more extensive

G 2

than that of Plana; and the numerical evaluation of every term is more complete than that of any preceding writer. Some terms to which we should have attached great interest are lost (at least for the present) by the untimely death of M. Delaunay.

Now, in all these works, so far as I have remarked, the following characteristics hold:—

(1). Each investigator has begun his work *de novo*, without making any use of the results of preceding investigators, even with the application of contingent corrections.

(2). Each investigator has used the fractions, in symbolical terms, to which I have alluded; and, by adherence to the symbolical form, has been compelled to expand them in series with rapidly increasing coefficients.

(3). The nature of the steps has compelled the investigators to decide the succession of their terms, not by numerical magnitude, but by algebraical order. And this has produced great inequality of convergence. Delaunay's smaller coefficients are probably correct, as he has exhibited them, to $0''\cdot0001$; but his larger terms converge so slowly that he has been compelled to supplement them by an assumed law of decrease; and they may perhaps be in error by almost $1''\cdot0000$.

(4). The mental labour in these operations is fearfully great. M. Plana once remarked to me, "Quelquefois, Monsieur, ces calculs me font presque perdre la tête."

(5). This labour cannot be alleviated, even in the examination of work done, by an amanuensis or assistant.

In consideration of these circumstances (which I have known, as well from examination of the works of others, as from my private investigations), I have long held the opinion that a Lunar Theory, in which every coefficient is expressed, from the very beginning of the process and throughout, by simple numbers, is very desirable. My ideas on this subject have by degrees assumed an orderly form; and I am now able to exhibit their leading points, as follow:—

(1*). I propose to assume Delaunay's final numerical expressions, for longitude, latitude, and parallax, with the addition of secular equations, as my fundamental numbers. These will be converted into other numerical expressions referred to more convenient units. To every number, as far as I think necessary, will be attached a symbolic term for contingent correction; in some cases considered as varying with the time. In all cases I assume that this correction will be so small that its first power will be sufficient. The secular terms will probably introduce cosines with sines of the same argument.

(2*). I propose to substitute these numbers with symbolical corrections in the equations in which the time is adopted as independent variable. The fractions to

which I have alluded will still occur, but not in a troublesome symbolical form. The greatest complication of denominators will be that of "a number with small symbolical correction attached to it;" which will be instantly converted into two terms without denominator. There will never be an infinite series.

(3*). The order of terms will be numerical; and, as far as I perceive, they will be equally accurate throughout.

(4*). The details of work will be very easy.

(5*). A great part of the work can be intrusted to a mere computer; and probably the whole can be examined, or can be repeated in duplicate, by such assistant.

To these I add,

(6*). I have strong confidence that equations of very long period may thus be examined with great severity, especially when there is reason to suspect that the form of the principal arguments may be slightly changed.

(7*). The result of the comparison of the terms in the mechanical or gravitational equations will be, a great number of equations for determining the numerical values of a great number of small quantities. I anticipate no difficulty in the solution; it is usually sufficient, for the determination of any one of the small quantities, to change (where necessary) the sign of its coefficient, so as to have all its coefficients with the same sign (the sign of the constant term being also changed), and to add all; neglecting all the other unknown quantities. In some cases, however, it may be necessary to treat two of these corrections in combination.

Though very late, I have actually begun a Lunar Theory in the shape which I have described. It is sufficiently possible that I may not be able to complete it; but I desire to leave it in such a state that a successor may be able to take it up successfully. For this purpose, I will enter into some further details as to the steps which I have made.

I. I refer all ordinates to an invariable plane and an invariable line in that plane; for instance, the plane of the ecliptic and the equinoxial line for the beginning of the year 1900.

II. I represent the masses of the Sun, the Earth, and the Moon, by the Greek letters σ, ϵ, μ. It is supposed that they are estimated, as is usual, by the acceleration which their attraction at distance 1 would produce in time 1. The letter σ' is accented to show that it is necessary to apply a term of contingent correction to the assumed Sun's mass, and similarly for ϵ' and μ'.

III. It is easily demonstrated that, to a very high degree of accuracy, the motion of the centre of gravity of Earth and Moon is subject to the same laws as the motion of a planet in that place, and that their action on Sun and planets is the same as if their mass were collected in that place.

IV. Explaining at present only a small part of the adopted

notation: A′, R′, V′, are the true major axis of orbit, true radius vector, and true longitude, of the 'centre of gravity of Earth and Moon,' measured from the Sun; a', r', v', those of the Moon as measured from the Earth; $v_{,}$, the tropical longitude of the Moon; l′, the latitude of the Moon. The perturbing forces are P, measured from the Earth in the projection of the Moon's radius vector on the fixed plane; T, at right angles to P in that plane, accelerating the tangential motion; and Z, measured from that plane and at right angles to it.

V. The three gravitational equations of motion are the following: in which the left-hand-side contains nothing but ordinates and forces produced by the Earth and Moon considered as points, and the right-hand-side contains nothing but forces produced by the Earth's oblateness and the Sun's disturbance (applicable in the same shape to a planet's disturbance):—

$$\frac{a'^3}{\epsilon'+\mu'}\cdot\left(\frac{a}{a'}\right)^2\cdot\frac{d}{dt}\left\{\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos \mathrm{l}'\right)^2\cdot\frac{dv'}{dt}\right\} = \frac{a'^2}{\epsilon'+\mu'}\left(\frac{r'}{a'}\cos \mathrm{l}\right)\cdot \mathrm{T}.$$

$$\frac{a'^3}{\epsilon'+\mu'}\cdot\frac{d}{dt}\left\{\left(\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos \mathrm{l}'\right)\right)^2+\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos \mathrm{l}'\cdot\frac{dv'}{dt}\right)^2\right\}$$

$$+\,2\,\frac{a'}{a}\cdot\left(\frac{a'}{r'}\right)^2\cos \mathrm{l}'\cdot\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos \mathrm{l}'\right)$$

$$=2\,\frac{a'^3}{\epsilon'+\mu'}\,\frac{\mathrm{P}}{a}\cdot\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos \mathrm{l}'\right)+2\,\frac{a'^3}{\epsilon'+\mu'}\cdot\frac{\mathrm{T}}{a}\cdot\frac{a'}{a}\cdot\frac{r'}{a'}\cos \mathrm{l}'\frac{dv'}{dt}.$$

$$\frac{a'^3}{\epsilon'+\mu'}\cdot\frac{d^2}{dt^2}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cdot\sin \mathrm{l}\right)+\frac{a'}{a}\left(\frac{a'}{r'}\right)^2\sin \mathrm{l}=\frac{a'^3}{\epsilon'+\mu'}\cdot\frac{\mathrm{Z}}{a}.$$

VI. For developing the left-hand-sides by substitution of Delaunay's values (subsequently to be furnished with additional symbols of contingent corrections), Delaunay's coefficients are to be converted into simple numbers: those for the powers of $\frac{r}{a}$ or $\frac{a}{r}$ into fractions of unity in which the last retained unit is the ten-millionth part of unity or 10^{-7}; and those for the terms of longitude and latitude into fractions of radius in which the last-retained unit is the ten-millionth part of radius (corresponding very nearly to 0″·02). The decimal cyphers preceding the efficient figures are to be omitted.

VII. For developing the right-hand-sides, it will be necessary to adopt for the last-retained unit the thousand-millionth part of unity or radius, in order to secure the terms which rise greatly by integration. But this amounts practically to the same extent of development as that already mentioned, because two decimal places are supplied by the smallness of the external factor representing the Sun's disturbing force (the value of its principal term being about $\frac{1}{180}$).

VIII. The following appears to be the expression for the right-hand-side of the first equation; where e is the ellipticity of the Earth's surface; $\frac{eq.\ c.\ f.}{grav.}$ is the proportion of centrifugal force at the equator to gravity; c is the Earth's polar semi-axis, and ω is the inclination of the equator to the ecliptic:—

$$+\left[\left(\mathrm{e}-\frac{eq.\,c.\,f.}{2\,grav.}\right)\cdot\left(\frac{c}{a'}\right)^2\right]\left(\frac{a'}{r'}\right)^3\cdot\left\{-2\sin^2\omega\cdot\cos^2\mathrm{l}\cdot\sin v_{,}\cdot\cos v_{,}\right.$$
$$\left.-2\cos\omega\cdot\sin\omega\ \cos\mathrm{l}\cdot\sin\mathrm{l}\cdot\cos v_{,}\right\}$$

$$+\left[\frac{\sigma'.a'^3}{(\epsilon'-\mu')\mathrm{A}'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\left(\frac{\mathrm{A}'}{\mathrm{R}'}\right)^3\cdot\left(\frac{r'}{a'}\right)^2\cdot\left\{-3\cos^2\mathrm{l}\cdot\cos\overline{v'-\mathrm{V}'}\cdot\sin\overline{v'-\mathrm{V}'}\right\}$$

$$+\left[\frac{\sigma'.a'^3}{(\epsilon'+\mu')\mathrm{A}'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu)^2}\cdot\frac{a'}{\mathrm{A}'}\right]\cdot\left(\frac{\mathrm{A}'}{\mathrm{R}'}\right)^4\cdot\left(\frac{r'}{a'}\right)^3\cdot\left\{+\frac{15}{2}\cos^3\mathrm{l}\cdot\cos^2\overline{v'-\mathrm{V}'}\cdot\sin\overline{v'-\mathrm{V}'}\right.$$
$$\left.-\frac{3}{2}\cos\mathrm{l}\cdot\sin\overline{v'-\mathrm{V}'}\right\}$$

$$+\left[\frac{\sigma'.a'^3}{(\epsilon'+\mu')\mathrm{A}'^3}\cdot\frac{\epsilon^3+\mu^3}{(\epsilon+\mu)^3}\cdot\left(\frac{a'}{\mathrm{A}'}\right)^2\right]\left(\frac{\mathrm{A}'}{\mathrm{R}'}\right)^5\cdot\left(\frac{r'}{a'}\right)^4\left\{-\frac{35}{2}\cos^4\mathrm{l}\cdot\cos^3\overline{v'-\mathrm{V}'}\cdot\sin\overline{v'-\mathrm{V}'}\right.$$
$$\left.+\frac{15}{2}\cdot\cos^2\mathrm{l}\cdot\cos\overline{v'-\mathrm{V}'}\cdot\sin\overline{v'-\mathrm{V}'}\right\}$$

$$+\left[\frac{\sigma'.a'^3}{(\epsilon'+\mu')\mathrm{A}'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\left(\frac{\mathrm{A}'}{\mathrm{R}'}\right)^3\cdot\left(\frac{r'}{a'}\right)^2\cdot bt\left\{-3\cos\mathrm{l}\cdot\sin\mathrm{l}\cdot\sin\overline{v'-\mathrm{V}'}\cdot\sin\overline{\mathrm{V}'-\mathrm{K}}\right\}$$

$$+\left[\frac{\sigma'.a'^3}{(\epsilon'+\mu')\mathrm{A}'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu^2}\cdot\frac{a'}{\mathrm{A}'}\right]\cdot\left(\frac{\mathrm{A}'}{\mathrm{R}'}\right)^4\cdot\left(\frac{r'}{a'}\right)^3\cdot bt$$
$$\times\left\{+15\cos^2\mathrm{l}\cdot\sin\mathrm{l}\cdot\cos\overline{v'-\mathrm{V}'}\cdot\sin\overline{v'-\mathrm{V}'}\cdot\sin\overline{\mathrm{V}'-\mathrm{K}}\right\}$$

The first line contains the effect of the Earth's oblateness; the second line contains the solar perturbations which first present themselves; the third line contains the principal parallactic terms; the fourth line, succeeding terms of smaller magnitude; the fifth and sixth lines, terms depending on the secular change in the position of the ecliptic, whose effect on the heliocentric latitude of the 'centre of gravity of Earth and Moon' is supposed to be represented by the term $bt \sin \overline{\mathrm{V}'-\mathrm{K}}$

A general periodical term may be added, to represent perturbation produced by any unrecognised cause.

IX. The right-hand-side of the second equation appears to consist of the following terms:—

$$+2\left[\left(e-\frac{eq.\,c.\,f.}{2\,grav.}\right)\left(\frac{c}{a}\right)^2\right]\left(\frac{a'}{r'}\right)^4\cdot\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos l'\right)$$

$$\times\left\{\begin{array}{l}5\,(\sin\omega\cdot\cos l\cdot\sin v_{,}+\cos\omega\cdot\sin l)^2\cdot\cos l\\ -2\,(\sin\omega\cdot\cos l\cdot\sin v_{,}+\cos\omega\cdot\sin l)\cdot\sin\omega\cdot\sin v_{,}\\ -\cos l\end{array}\right\}$$

$$+2\left[\frac{\sigma'.a'^3}{(\epsilon'+\mu').A'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\cdot\left(\frac{A'}{R'}\right)^3\frac{r'}{a'}\cdot\frac{d}{dt}\left(\frac{a'}{a}\,\frac{r'}{a'}\cos l'\right)$$

$$\times\left\{+3\cos l'\cdot\cos^2\overline{v'-V'}-\cos l'\right\}$$

$$+2\left[\frac{\sigma'.a'^3}{(\epsilon'+\mu').A'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu)^2}\cdot\frac{a'}{A'}\right]\left(\frac{A'}{R'}\right)^4\cdot\left(\frac{r'}{a'}\right)^2\cdot\frac{d}{dt}\left(\frac{a'}{a}\,\frac{r'}{a'}\cos l'\right)$$

$$\times\left\{-\frac{15}{2}\cos^2 l'\cdot\cos^3\overline{v'-V'}+\frac{3}{2}\cos\overline{v'-V'}+3\cos^2 l'\cdot\cos\overline{v'-V'}\right\}$$

$$+2\left[\frac{\sigma'.a'^3}{(\epsilon'+\mu').A'^3}\cdot\frac{\epsilon^3+\mu^3}{(\epsilon+\mu)^3}\cdot\left(\frac{a'}{A'}\right)^2\right]\cdot\left(\frac{A'}{R'}\right)^5\cdot\left(\frac{r'}{a'}\right)^3\cdot\frac{d}{dt}\left(\frac{a'}{a}\cdot\frac{r'}{a'}\cos l'\right)$$

$$\times\left\{+\frac{35}{2}\cos^3 l\cdot\cos^4\overline{v'-V'}-\frac{15}{2}\cos l\cdot\cos^2\overline{v'-V'}\right.$$

$$\left.-\frac{15}{2}\cos^3 l\cdot\cos^2\overline{v'-V'}+\frac{3}{2}\cos l\right\}$$

$$+2\left[\frac{\sigma'.a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\cdot\left(\frac{A'}{R'}\right)^3\cdot\frac{r'}{a'}\cdot\frac{d}{dt}\left(\frac{a'}{a}\,\frac{r'}{a'}\cos l'\right)$$

$$\times bt\cdot\left\{+3\sin l'\cdot\cos\overline{V'-K}\,\sin\overline{V'-K}\right\}$$

$$+2\left[\frac{\sigma'.a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu)^2}\cdot\frac{a'}{A'}\right]\cdot\left(\frac{A'}{R'}\right)^4\cdot\left(\frac{r'}{a'}\right)^2\cdot\frac{d}{dt}\left(\frac{a'}{a}\,\frac{r'}{a'}\cos l'\right)$$

$$\times bt\left\{-15\cdot\cos l'\cdot\sin l'\cdot\cos^2\overline{v'-V'}\cdot\sin\overline{V'-K}+3\cos l'\cdot\sin l'\cdot\sin\overline{V'-K}\right\}$$

$$+2\frac{dv'}{dt}\times\left\{\text{all the terms of Article VIII.}\right\}$$

X. The right-hand-side of the third equation appears to consist of the following terms:—

$$+\left[\left(e-\frac{eq.\,c.\,f.}{2\,grav.}\right)\left(\frac{c}{a}\right)^2\right]\left(\frac{a'}{r'}\right)^4$$

$$\times\left\{\begin{array}{l}5\,(\sin\omega\cdot\cos l.\sin v_{,}+\cos\omega\cdot\sin l)^2\times\sin l-2.\sin\omega\cdot\cos\omega\cdot\cos l.\sin v_{,}\\ -\sin^2\omega.\sin l-3\cos^2\omega\cdot\sin l\end{array}\right\}$$

$$+\left[\frac{\sigma' a'^3}{(\epsilon'+\mu')\cdot A'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\left(\frac{A'}{R'}\right)^3\cdot\frac{r'}{a'}\cdot\left\{-\sin 1\right\}$$

$$+\left[\frac{\sigma' a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu)^2}\cdot\frac{a'}{A'}\right]\cdot\left(\frac{A'}{R'}\right)^4\cdot\left(\frac{r'}{a'}\right)^2\left\{+3\cdot\cos 1\cdot\sin 1\cdot\cos.\overline{v'-V'}\right\}$$

$$+\left[\frac{\sigma' a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^3+\mu^3}{(\epsilon+\mu)^3}\cdot\left(\frac{a'}{A'}\right)^2\right]\cdot\left(\frac{A'}{R'}\right)^5\cdot\left(\frac{r'}{a'}\right)^3$$

$$\left\{-\frac{15}{2}\cos^2 1\cdot\sin 1\cdot\cos^2\overline{v'-V'}+\frac{3}{2}\sin 1\right\}$$

$$+\left[\frac{\sigma' a'^2}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon+\mu}{\epsilon+\mu}\right]\left(\frac{A'}{R'}\right)^3\cdot\frac{r'}{a'}\cdot bt\left\{+3\cos 1\cdot\cos\overline{v'-V'}\cdot\sin\overline{V'-K}\right\}$$

$$+\left[\frac{\sigma' a'^3}{(\epsilon'+\mu')A'^3}\cdot\frac{\epsilon^2-\mu^2}{(\epsilon+\mu)^2}\cdot\frac{a'}{A'}\right]\cdot\left(\frac{A'}{R'}\right)^4\cdot\frac{r'^2}{a'}\cdot bt$$

$$\times\left\{\begin{array}{l}-\frac{15}{2}\cos^2\lambda\cdot\cos^2\overline{v'-V'}\cdot\sin\overline{V'-K}\\ +\frac{3}{2}\sin\overline{f-V'}+3\sin^2 1\cdot\sin\overline{V'-K}\end{array}\right\}$$

All these algebraical expressions require revision, and may perhaps admit of some reduction.

XI. The nature of the considerations which will introduce some of the contingent variations may be judged partly from discussions similar to the following:—

Assuming that the length of the mean sidereal year is invariable, and is accurately known, $\frac{\sigma'}{A'^3}$ is invariable, and is a known quantity. But A′ is not certainly known; and, therefore, if A be its assumed value, we must write its real value in some such shape as this: $A' = A\left(1 + (1)\right)$, the quantity (1) being a contingent correction. Then, as $\frac{\sigma'}{A'^3}$ is invariable, or $= \frac{\sigma}{A^3}$, $\sigma' = \sigma\frac{A'^3}{A^3} = \sigma\left(1 + 3\,(1)\right)$.

Now we cannot assume that the length of the mean sidereal month is invariable; moreover, our estimate of the masses $\epsilon + \mu$ may be liable to error. Therefore, a' ought to be expressed by $a\left(1 + (2)\right)$, where a is the assumed value for 1900, and where (2) contains a constant value, and also a value depending on time. And $\epsilon' + \mu'$ must be represented by $\left(\epsilon+\mu\right)$

$\left(1+(3)\right)$, where (3) is constant. Therefore, $\frac{\sigma' a'^3}{(\varepsilon'+\mu')A'^3}$ will become $\frac{\sigma a^3}{(\varepsilon+\mu)A^3} \times \left(1+3(1)\right) \cdot \left(1+3(2)\right) \cdot \left(1-(3)\right) \cdot \left(1-3(1)\right) = \frac{\sigma a^3}{(\varepsilon+\mu)A^3} \times \left(1+3(2)-(3)\right)$. And to form $\frac{\sigma' a'^3}{(\varepsilon'+\mu')A'^3} \cdot \frac{a'}{A'}$, we must multiply the last expression by $\frac{a}{A}\left(1+(2)-(1)\right)$, and it becomes $\frac{\sigma a^3}{(\varepsilon+\mu)A^3} \cdot \frac{a}{A} \times \left(1-(1)+4(2)-(3)\right)$.

No general rule can be given for the appropriation of contingent symbols to terms. But every principal coefficient (as the coefficients of elliptic equation, evection, variation, annual equation, inclination, evection in polar distance, and many much smaller coefficients,) must have its independent symbol of contingent correction.

The first part of the process of applying these principles to the Lunar Theory will consist in developing the expressions of Articles VIII., IX., X., by substitution in them of Delaunay's numerical values of $\frac{a}{r}$, v, l, and in ascertaining how nearly these values will satisfy the three equations of Article V.

I have developed, numerically, the expressions for all the requisite powers of $\frac{r}{a}$ and $\frac{a}{r}$, and I am proceeding with the development of sin l and various powers of cos l.

1874, *January* 9.

Additional Notes concerning Sir William Herschel's Double Stars. By S. W. Burnham, Esq.

(*Communicated by Mr. Dunkin.*)

I have given some attention, more particularly within the last few months, to the telescopic examination of Double Stars with reference to which there exists some uncertainty as to place, magnitude, position-angle, and distance, identity with other double stars, &c. There is a large number of such objects, several hundred perhaps, principally from the Catalogues of the two Herschels, which require or may require correction in some of these respects. Some have been found to be out of their